Minimization of Systemic Risk as an Optimal Network Reorganization Problem

The Case of Overlapping Portfolio Networks in the European Government Bond Market

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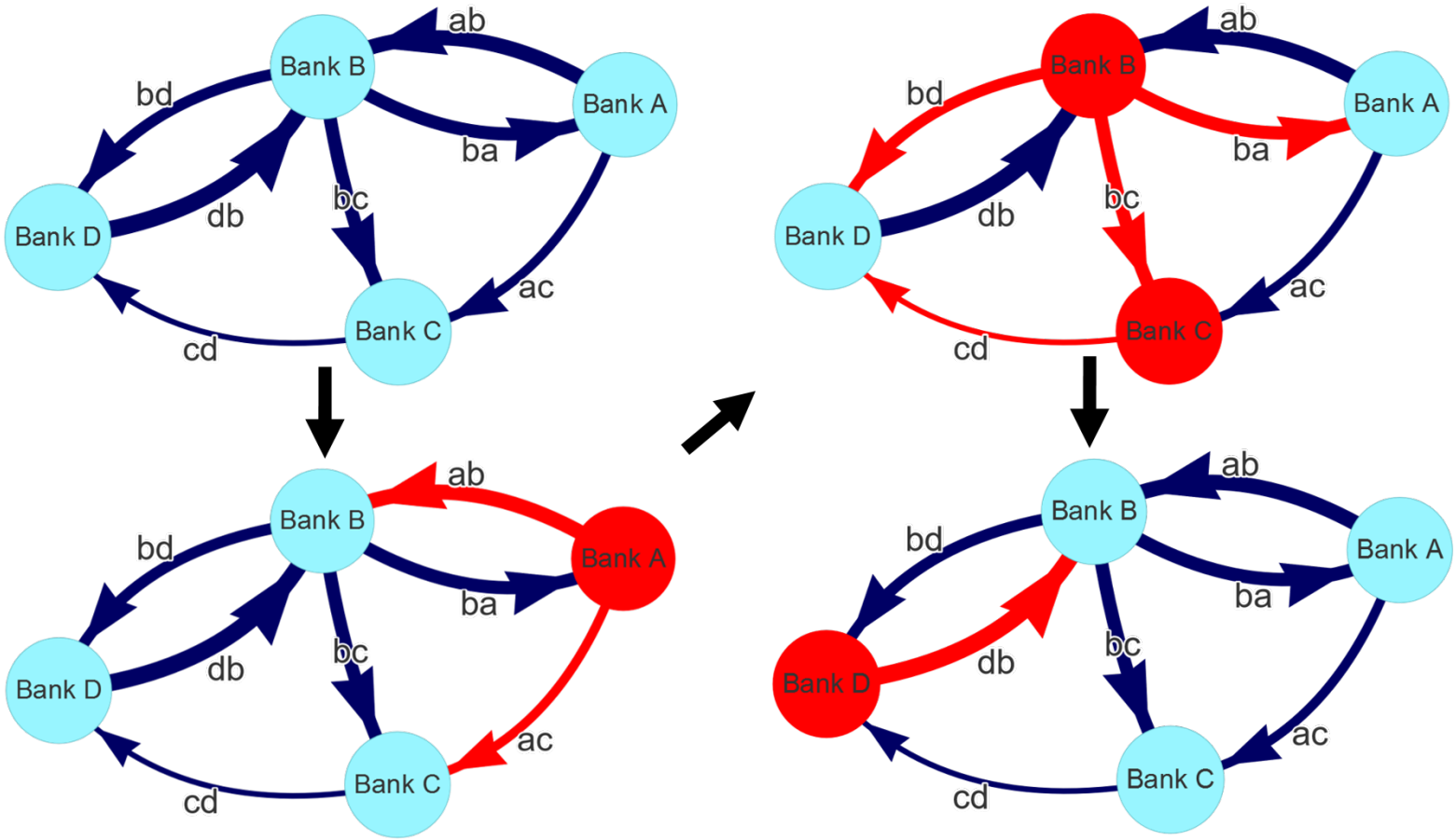
COMPLEXITY SCIENCE HUB VIENNA

Motivation

- Financial systems are multi-layer networks $\rightarrow$ need all layers to understand SR
- Layer of overlapping portfolios relatively little investigated
- Can we think of optimally allocated markets? How do they look like?

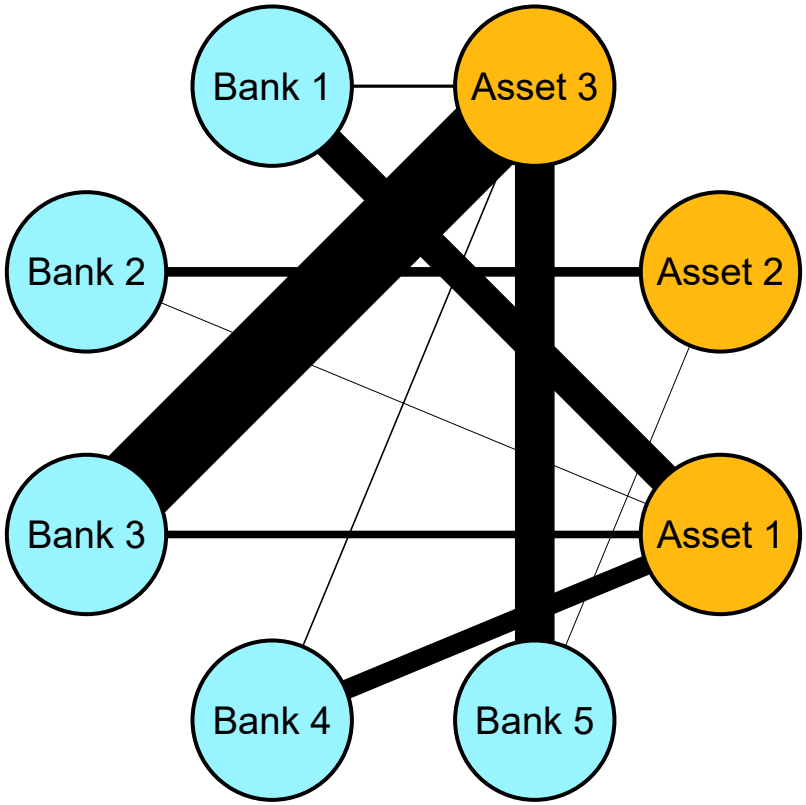
Measuring Systemic Risk - DebtRank



Battiston et al. (2012), Thurner and Poledna (2013)

Overlapping Portfolios as Bipartite Network

Example: $N = 5,\ K = 3$



Simple one-mode projection does not work out

Model - Liquidity-Adjusted One-Mode Projection

- Assume simple linear price impact wrt to market depth D_k
- ADV_k ... average daily volume

$$\Delta p_k(z) = \alpha \frac{z}{D_k}, \qquad D_k = c \frac{ADV_k}{\sigma_k}$$

If bank i sells V_{ki} of asset k, price is depressed by $\frac{V_{ki}}{D_k}$ (max. impact)
If bank j has V_{kj} of asset k, experience loss of $V_{kj}\frac{V_{ki}}{D_k}$

$$w_{ij} = \sum_{k=1}^{K} V_{kj} V_{ki} \frac{1}{D_k} \qquad \text{or in matrix form} \qquad w = V^\top D^{-1} V$$

Cont and Schaanning (2017), Braverman and Minca (2014), Guo et al. (2016)

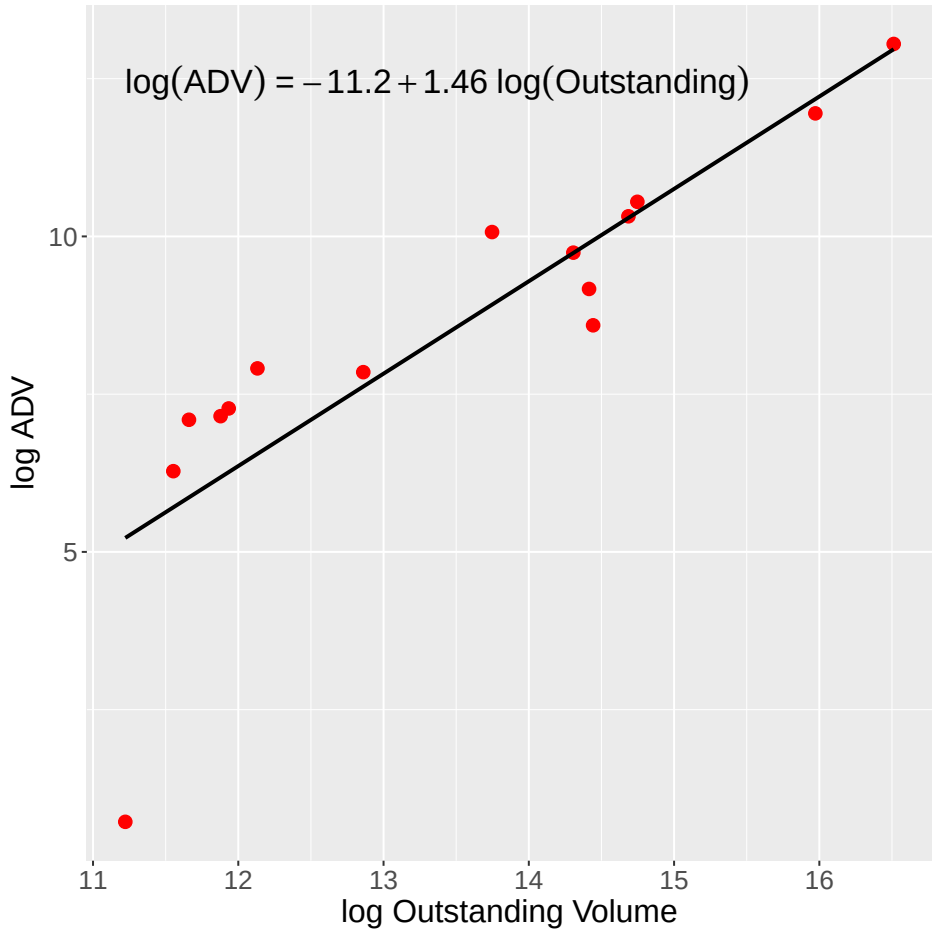
Data

- European Banking Authority (EBA)
- Stress test data from 2016
- 51 major European banks (49 included in analysis)
- 44 sovereign bond investment categories (36 included)

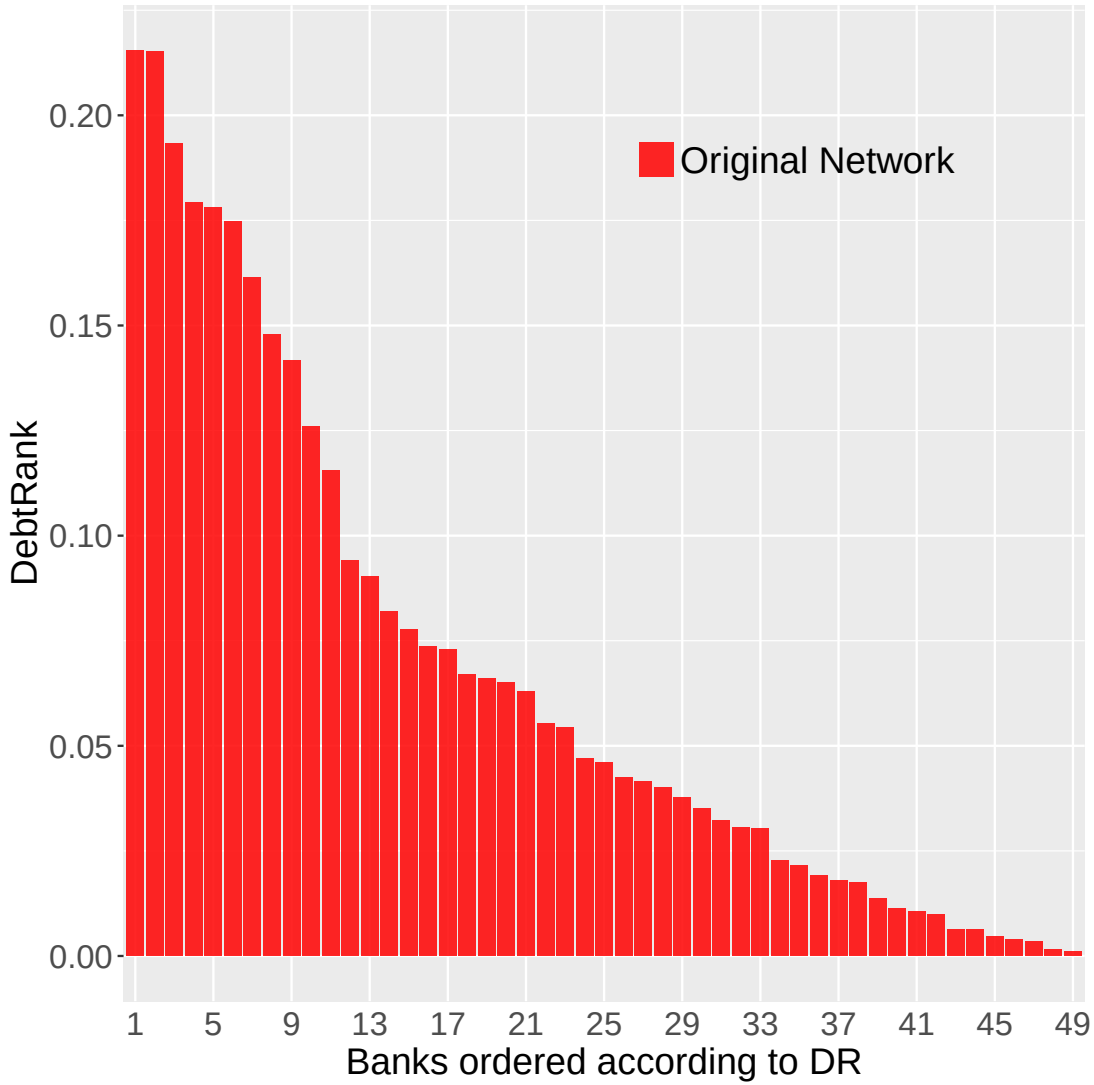
European Government Bond Network

Estimating Market Depth

$$\log(ADV) = b_0 + b_1 \log(Outstanding) + \xi$$



SR in the European Government Bond Network



Reducing an Externality

SR of Market

$$\bar{R} = \frac{1}{N} \sum_{i=1}^{N} R_i$$

- Keep portfolio volumes and market decomposition constant
- Measure 'risk profile' of portfolio by Markowitz mean-variance approach
- Focus on direct impacts

Formulating a SR-Minimization problem

$$
\begin{aligned}
\min_{x_{ki} \geq 0 \ \forall k,i} \quad & f(x) = \sum_i \sum_j \frac{1}{E_j} \sum_k x_{ki} x_{kj} \frac{1}{D_k} \\
\text{subject to} \quad & V_i = \sum_k x_{ki}, \quad \forall i, \\
& S_k = \sum_i x_{ki}, \quad \forall k, \\
& \tilde{r}_i \leq \sum_k x_{ki} r_k, \quad \forall i, \\
& \tilde{\sigma}_i^2 \geq \sum_k \sum_l x_{ki} x_{li} \sigma_{kl}^2, \quad \forall i,
\end{aligned}
\tag{1}
$$

This is a QCQP

$$\begin{aligned} \min_{y \geq 0} \quad & \frac{1}{2} y^\top (P_0^\top + P_0) y \\ \text{subject to} \quad & y^\top P_1 y + c_1 \leq 0 \\ & A_1 y + c_2 \leq 0 \\ & A_2 y + c_3 = 0 \end{aligned} \tag{2}$$

- $y = vec(X)$ is the vectorization of matrix X
- P_0 and P_1 are $(KN \times KN)$-matrices
- A_1 a $(N \times KN)$-matrix, A_2 a $\{(K+N) \times KN\}$-matrix
- c_1, c_2, c_3 are KN-dimensional vectors

- Difficulty of finding solution depends on definiteness of $P_0^\top + P_0$

Solving the Problem

NEOS Server: State-of-the-Art Solvers for Numerical Optimization

The **NEOS Server** is a free internet-based service for solving numerical optimization problems. Hosted by the Wisconsin Institute for Discovery at the University of Wisconsin in Madison, the NEOS Server provides access to more than 60 state-of-the-art solvers in more than a dozen optimization categories. Solvers hosted by the University of Wisconsin in Madison run on distributed high-performance machines enabled by the HTCondor software; remote solvers run on machines at Argonne National Laboratory, Arizona State University, the University of Klagenfurt in Austria, and the University of Minho in Portugal.

Extended Mathematical Programming –

- DE [GAMS Input]
- JAMS [GAMS Input]

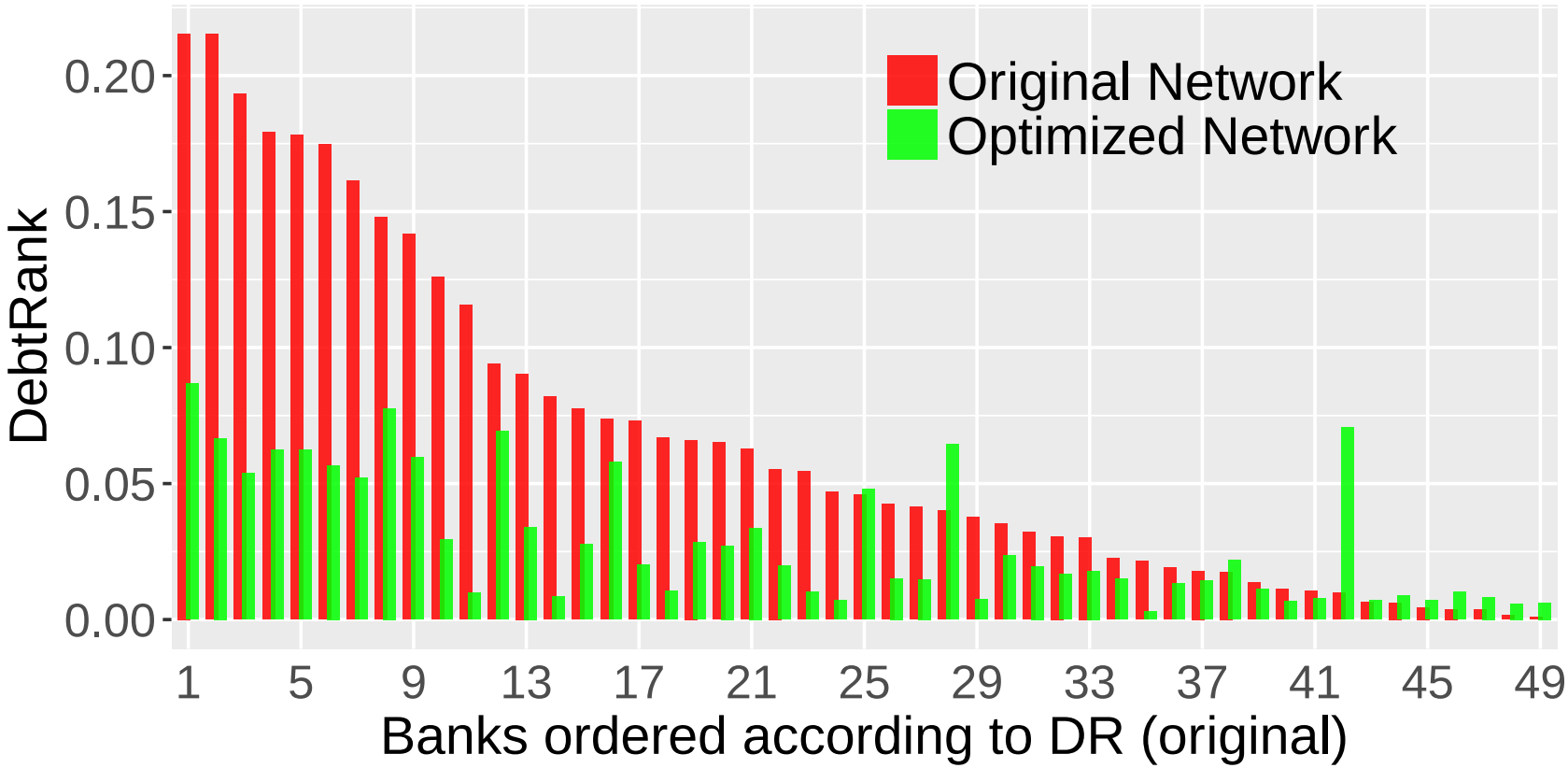
Global Optimization –

- ASA [AMPL Input]
- BARON [AMPL Input][GAMS Input]
- Couenne [AMPL Input][GAMS Input]
- icos [AMPL Input]
- LINDOGlobal [GAMS Input]
- PGAPack [AMPL Input]
- PSwarm [AMPL Input]
- scip [AMPL Input][CPLEX Input][GAMS Input][MPS Input][OSIL Input][ZIMPL Input]

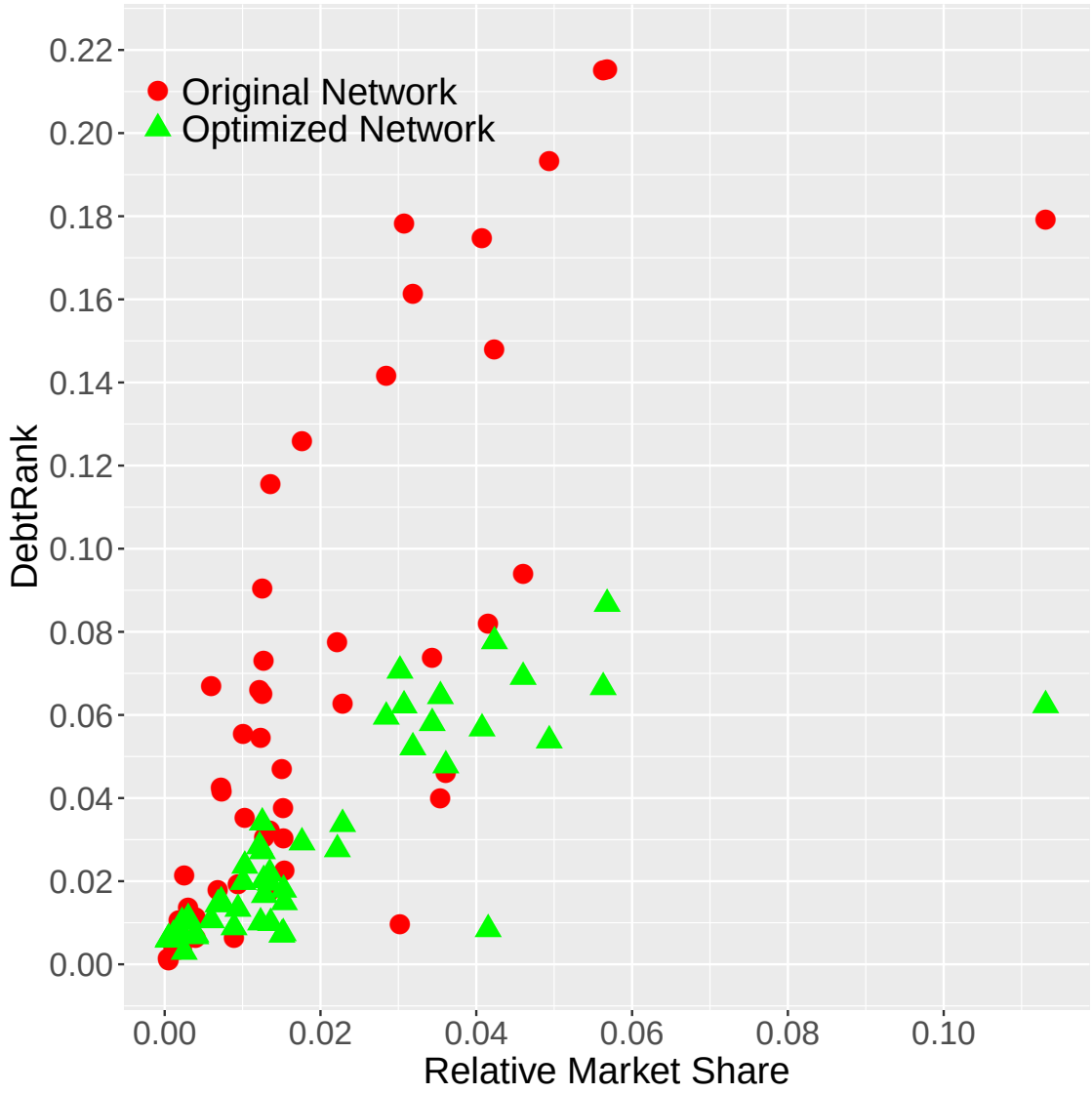
Linear Network Programming –

- RELAX4 [DIMACS Input][RELAX4 Input]

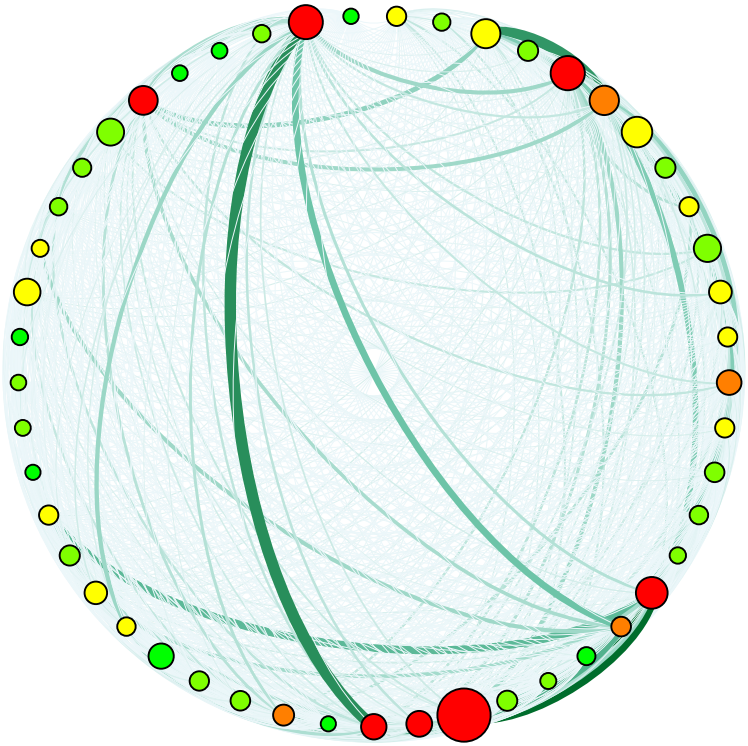
Results



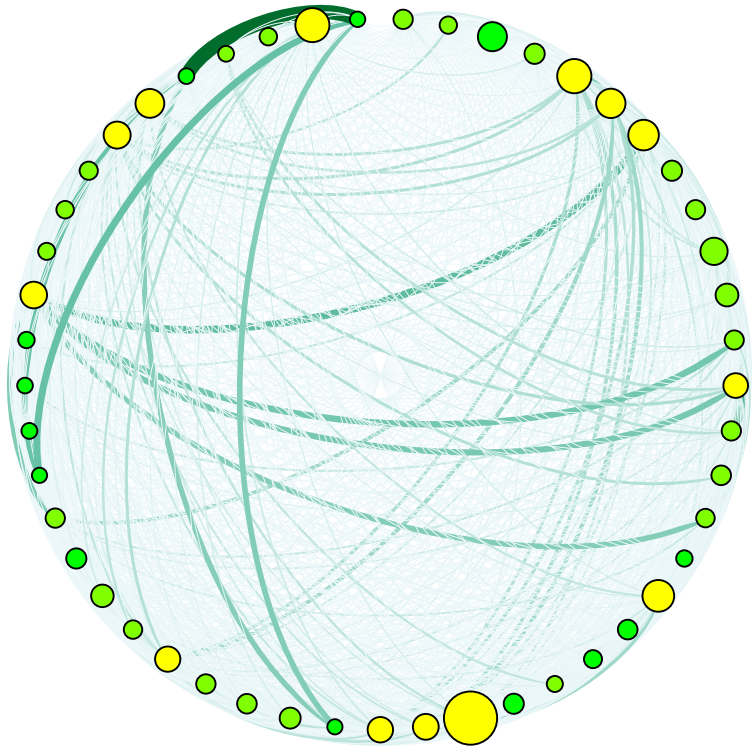
- $\bar{R}^{orig} = 6.67\%$
- $\max R_i^{orig} = 21.54\%$
- $\bar{R}^{optim} = 2.89\%$
- $\max R_i^{optim} = 8.67\%$

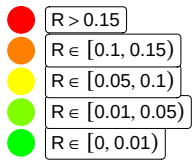

Original Network
Optimized Network
DebtRank
Relative Market Share
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20
0.22

Original Network



Optimized Network





Discussion

- We can reduce SR by reshuffling financial network without changing banks' investment strategies
- Optimization yields a theoretical benchmark
- Optimize direct exposure networks (e.g. interbank liabilities) wrt. SR?

References

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